応用数学 A 演習 No. 9 解答

問題 1 (1) $\left\|\frac{1}{\sqrt{2\pi}}\right\|^2 = \int_{-\pi}^{\pi} \left(\frac{1}{\sqrt{2\pi}}\right)^2 dx = \int_{-\pi}^{\pi} \frac{1}{2\pi} dx = \frac{1}{2\pi} \times 2\pi = 1$

(2) $\left\|\frac{1}{\sqrt{\pi}}\cos nx\right\|^2 = \int_{-\pi}^{\pi} \left(\frac{1}{\sqrt{\pi}} \cos nx\right)^2 dx = \int_{-\pi}^{\pi} \frac{1}{\pi} \cos^2 nx\, dx$

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ だから

$$= \frac{1}{\pi} \int_{-\pi}^{\pi} \frac{1+\cos 2nx}{2} dx = \frac{1}{\pi} \left[\frac{x}{2} + \frac{\sin 2nx}{4n}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi} \left\{\left(\frac{\pi}{2} + \frac{\sin 2n\pi}{4n}\right) - \left(\frac{-\pi}{2} + \frac{\sin(-2nx)}{4n}\right)\right\} = 1$$

(3) $\left(\frac{1}{\sqrt{2\pi}}, \frac{1}{\sqrt{\pi}}\cos nx\right) = \int_{-\pi}^{\pi} \frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{\pi}} \cos nx\, dx = \frac{1}{\sqrt{2}\pi} \int_{-\pi}^{\pi} \cos nx\, dx$

$$= \frac{1}{\sqrt{2}\pi} \left[\frac{1}{n}\sin nx\right]_{-\pi}^{\pi} = \frac{1}{\sqrt{2}n\pi}(\sin n\pi - \sin(-n\pi)) = 0.$$

(4) $\left(\frac{1}{\sqrt{\pi}}\cos nx, \frac{1}{\sqrt{\pi}}\sin mx\right) = \int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}}\cos nx \frac{1}{\sqrt{\pi}} \sin mx\, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx\, \sin mx\, dx$

ここで $\cos nx \sin mx$ は奇関数だから

$$= 0$$

積を和に直す公式を使ってもよい。

(5) $\left(\frac{1}{\sqrt{\pi}}\cos nx, \frac{1}{\sqrt{\pi}}\cos mx\right) = \int_{-\pi}^{\pi} \frac{1}{\sqrt{\pi}}\cos nx \frac{1}{\sqrt{\pi}} \cos mx\, dx = \frac{1}{\pi} \int_{-\pi}^{\pi} \cos nx\, \cos mx\, dx$

ここで $\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha+\beta)+\cos(\alpha-\beta))$ だから

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} (\cos(m+n)x + \cos(m-n)x)dx$$

$m+n \neq 0$, $m-n \neq 0$ だから

$$= \frac{1}{2\pi} \left[\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{\sin(m+n)\pi - \sin(-(m+n)\pi)}{m+n} + \frac{\sin(m-n)\pi - \sin(-(m-n)\pi)}{m-n}\right] =$$

0.