

## 電気数学演習 第6回 解答

1. (1)  $(1 + 5i) + (2 - 3i) = (1 + 2) + (5i - 3i) = 3 + 2i$

(2)  $(5 + 6i) - (3 - 5i) = (5 - 3) + (6i - (-5i)) = 2 + 11i$

(3)  $(-i)^5 = (-1)^5 i^5 = (-1)^5 i^2 i^2 i = (-1)^5 (-1)(-1)i = -i$

(4)  $(3 + 2i)(3 - 4i)$   
 $= 3 \times 3 + 3 \times (-4i) + (2i) \times 3 + (2i) \times (-4i)$   
 $= 9 - 12i + 6i + 8 = 17 - 6i$

(5)  $(3 + 2i)(3 - 2i)$   
 $= 3 \times 3 + 3 \times (-2i) + (2i) \times 3 + (2i) \times (-2i)$   
 $= 9 - 6i + 6i + 4 = 13$

(6)  $3i^3 = -3i$

(7)  $\frac{1-i}{1+i} = \frac{(1-i)(1-i)}{(1+i)(1-i)} = \frac{-2i}{1^2 - (i)^2} = \frac{-2i}{2}$   
 $= -i$

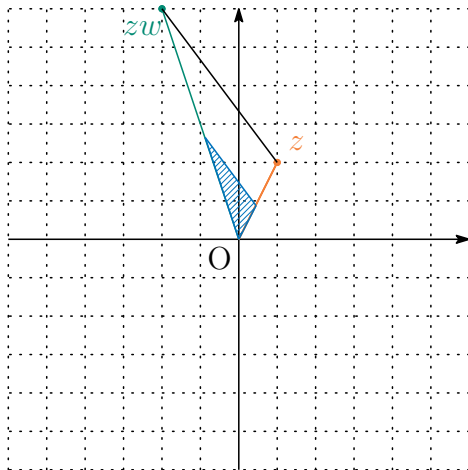
(8)  $\frac{2+3i}{4-5i} = \frac{(2+3i)(4+5i)}{(4-5i)(4+5i)} = \frac{-7+22i}{4^2 - (5i)^2} = \frac{-7+22i}{41}$   
 $= -\frac{7}{41} + \frac{22}{41}i$

(9)  $\frac{2}{1+i} = \frac{2(1-i)}{(1+i)(1-i)} = \frac{2(1-i)}{1^2 - (i)^2} = 1 - i$

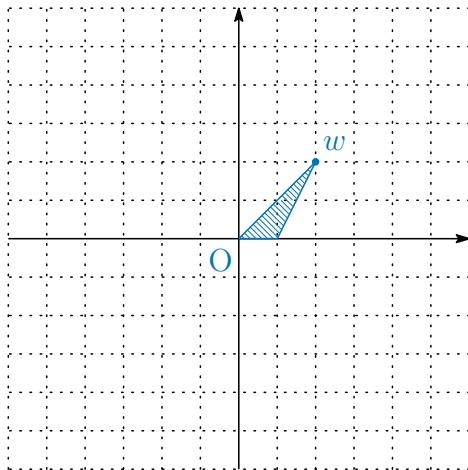
(10)  $\frac{1}{i} = \frac{1(-i)}{i(-i)} = -i$

2. 例えば  $z = 1 + 2i$ ,  $w = 2 + 2i$  とする.

(1)  $zw = -2 + 6i$  だから



(2)



(3) 例えば (2) の図中の三角形を写し取って (1) の三角形に当てはめてみよ。

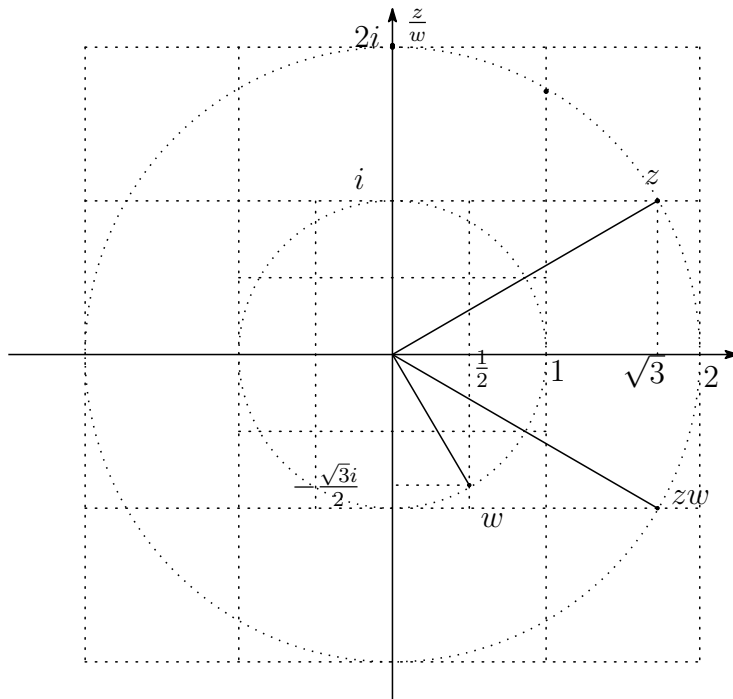
3  $i$  を虚数単位とし  $z = \sqrt{3} + i$ ,  $w = \frac{1}{2} - \frac{\sqrt{3}}{2}i$  とする.

(1)

$$zw = (\sqrt{3} + i) \left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) = \sqrt{3} - i$$

$$\frac{z}{w} = \frac{\sqrt{3} + i}{\frac{1}{2} - \frac{\sqrt{3}}{2}i} = \frac{(\sqrt{3} + i) \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right)}{\left( \frac{1}{2} - \frac{\sqrt{3}}{2}i \right) \left( \frac{1}{2} + \frac{\sqrt{3}}{2}i \right)} = 2i.$$

(2)  $z$ ,  $w$ ,  $zw$ ,  $\frac{z}{w}$  を複素平面上に図示せよ.



$$(3) |z| = \sqrt{\text{実部}^2 + \text{虚部}^2} = \sqrt{(\sqrt{3})^2 + (1)^2} = 2$$

$$\tan \arg z = \frac{1}{\sqrt{3}} \text{ で } z \text{ は第1象限にあるから } \arg z = \frac{\pi}{6}$$

$$|w| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\tan \arg w = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \text{ で } w \text{ は第4象限にあるから } \arg w = -\frac{\pi}{3}$$

等により

$$z = 2 \left( \cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)$$

$$w = 1 \left( \cos \left( -\frac{\pi}{3} \right) + i \sin \left( -\frac{\pi}{3} \right) \right)$$

$$zw = \sqrt{3} - i \text{ は}$$

$$|zw| = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$$

$$\tan \arg(zw) = \frac{-1}{\sqrt{3}} \text{ で } zw \text{ は第3象限にあるから } \arg(zw) = -\frac{\pi}{6}$$

以上から

$$zw = 2 \left( \cos \left( -\frac{\pi}{6} \right) + i \sin \left( -\frac{\pi}{6} \right) \right)$$

$$\frac{z}{w} = 2i \text{ は}$$

$$\frac{z}{w} = 2 \left( \cos \left( \frac{\pi}{2} \right) + i \sin \left( \frac{\pi}{2} \right) \right)$$

$$(4) |zw| = 2 = 2 \times 1 = |z||w|$$

$$\arg(zw) = -\frac{\pi}{6} = \frac{\pi}{6} + \left( -\frac{\pi}{3} \right) = \arg(z) + \arg(w)$$

$$\left| \frac{z}{w} \right| = 2 = \frac{2}{1} = \frac{|z|}{|w|}$$

$$\arg \frac{z}{w} = \frac{\pi}{2} = \frac{\pi}{6} - \left( -\frac{\pi}{3} \right) = \arg(z) - \arg(w)$$

で確かに回し伸ばしが成り立っている。