

電気のための線形代数 B
演習問題 No.6

学生番号

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6.1. (P)
$$\begin{cases} x_1 - x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 - 4x_3 = 1 \\ x_1 - 2x_2 + x_3 = 1 \end{cases}$$
 をクラメルの公式を用いて解きたい.

(1)

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ 1 & -2 & 1 \end{pmatrix}, \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

とおくと (P) $\Leftrightarrow A\vec{x} = \vec{b}$ となる.

$$\begin{aligned} (2) |A| &= \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ 1 & -2 & 1 \end{vmatrix} \begin{array}{l} \text{第1行を2倍して引く} \\ \text{第1行を引く} \end{array} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 5 & -10 \\ 0 & -1 & -2 \end{vmatrix} = 1 \times \begin{vmatrix} 5 & -10 \\ -1 & -2 \end{vmatrix} \\ &= 5 \times (-2) - (-1)(-10) = -10 - 10 = -20 \end{aligned}$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 3 & -4 \\ 1 & -2 & 1 \end{vmatrix} = - \begin{vmatrix} -1 & 3 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & -3 \\ 3 & -4 \end{vmatrix} = -(-1 + 6) + (4 - 9) = -5 - 5 = -10$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -4 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = (1 - 3) - (-4 - 6) = 8$$

$$\Delta_3 = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 1 & -2 & 1 \end{vmatrix} = - \begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = -(-2 + 1) + (3 + 2) = 1 + 5 = 6$$

(3) x_1, x_2, x_3 を求めよ.

$$x_1 = \frac{\Delta_1}{|A|} = \frac{1}{2}, x_2 = \frac{\Delta_2}{|A|} = -\frac{2}{5}, x_3 = \frac{\Delta_3}{|A|} = -\frac{3}{10}$$

追加

$$(P) \begin{cases} x_1 + 2x_2 + 3x_3 = 0 \\ x_1 + 3x_2 + 5x_3 = 0 \\ x_1 + 5x_2 + 12x_3 = 1 \end{cases} \text{ を解く。}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{vmatrix} = 3,$$

$$\Delta_1 = 1,$$

$$\Delta_2 = -2,$$

$$\Delta_3 = 1,$$

だから

$$x_1 = \frac{\Delta_1}{|A|} = \frac{1}{3}, x_2 = \frac{\Delta_2}{|A|} = -\frac{2}{3}, x_3 = \frac{\Delta_3}{|A|} = \frac{1}{3}$$