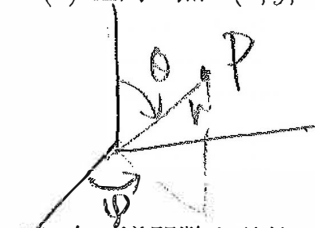


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1 (1) 空間の点 $P(x, y, z)$ の極座標を (r, θ, φ) とするとき, x, y, z を r, θ, φ で表せ.



$$\begin{cases} x = r \sin \theta \cos \varphi \\ y = r \sin \theta \sin \varphi \\ z = r \cos \theta \end{cases}$$

(2) 次の導関数を計算せよ.

$$x_r = \sin \theta \cos \varphi \quad x_\theta = r \cos \theta \cos \varphi \quad x_\varphi = -r \sin \theta \sin \varphi$$

$$y_r = \sin \theta \sin \varphi \quad y_\theta = r \cos \theta \sin \varphi \quad y_\varphi = r \sin \theta \cos \varphi$$

$$z_r = \cos \theta \quad z_\theta = -r \sin \theta \quad z_\varphi = 0$$

(3) この変換のヤコビアンを計算せよ.

$$\begin{vmatrix} \sin \theta \cos \varphi & r \cos \theta \cos \varphi & -r \sin \theta \sin \varphi \\ \sin \theta \sin \varphi & r \cos \theta \sin \varphi & r \sin \theta \cos \varphi \\ \cos \theta & -r \sin \theta & 0 \end{vmatrix}$$

$$= r^2 \sin^3 \theta \sin^2 \varphi + r^2 \sin \theta \cos^2 \theta \cos^2 \varphi$$

$$+ r^2 \sin \theta \cos^2 \theta \sin^2 \varphi + r^2 \sin^3 \theta \cos^2 \varphi$$

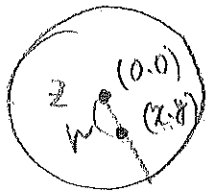
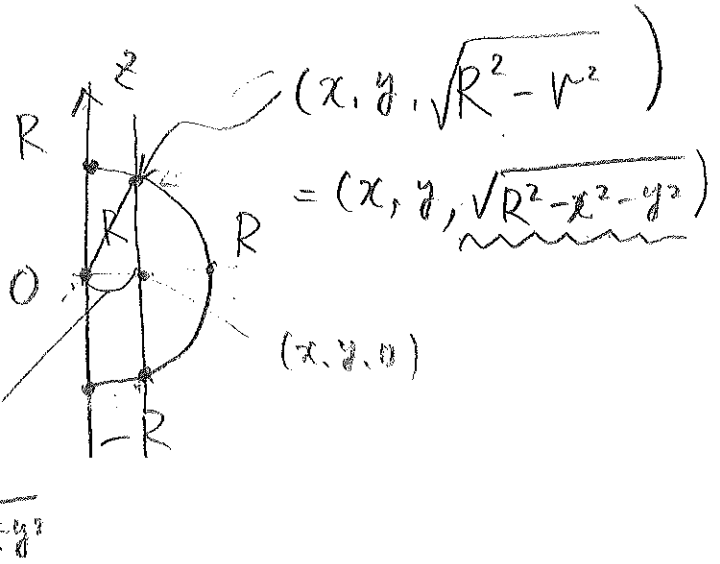
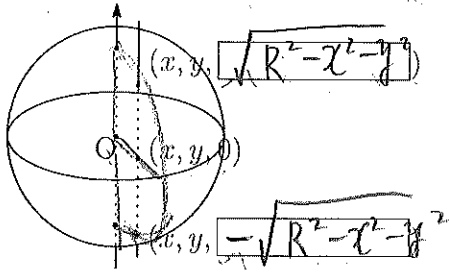
$$= r^2 \sin \theta \{ \sin^2 \theta \sin^2 \varphi + \cos^2 \theta \sin^2 \varphi \}$$

$$+ r^2 \sin \theta \{ \sin^2 \theta \cos^2 \varphi + \cos^2 \theta \cos^2 \varphi \}$$

$$= r^2 \sin \theta$$

2 G を原点中心半径 $R > 0$ の球とする。

(1) G の体積を串刺し法で計算せよ。



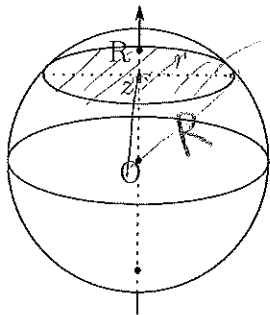
$$r = \sqrt{x^2 + y^2}$$

$$V = \iint_D 2\sqrt{R^2 - x^2 - y^2} dx dy = \text{あるいは } \int_0^R 2\pi r^2 dz$$

$$D = \{(x, y) \mid x^2 + y^2 \leq R^2\}$$

$$= \frac{4}{3} \pi R^3$$

(2) G の体積を輪切り法で計算せよ。



$$z^2 + r^2 = R^2 \text{ となる}$$

$$r = \sqrt{R^2 - z^2}$$

したがって断面の円の面積は

$$S(z) = \pi r^2 = \pi (R^2 - z^2)$$

$$\text{体積} = \int_{-R}^R S(z) dz = \pi \int_{-R}^R (R^2 - z^2) dz$$

$$= \pi \left[R^2 z - \frac{z^3}{3} \right]_{z=-R}^{z=R} = \pi \left(R^3 - \frac{R^3}{3} \right) - \pi \left(-R^3 + \frac{R^3}{3} \right)$$

$$= \frac{4}{3} \pi R^3$$