電気のための微分積分C　第2回解答

2.1.

(1) $\displaystyle\int (2x+6)\,dx = x^2+6x$

(2) (1) より

$$\int_0^5 (2x+6)\,dx = \Big[x^2+6x\Big]_0^5 = 55.$$

(3) $\displaystyle\int (2x+6)^4\,dx$

$2x+6=t$ とおくと $\dfrac{dt}{dx}=2$ すなわち $dx=\dfrac{dt}{2}$ だから

$$\int (2x+6)^4\,dx = \int t^4\,\frac{dt}{2} = \frac{1}{2}\int t^4\,dt = \frac{1}{2}\frac{t^5}{5} = \frac{1}{10}(2x+6)^5$$

(4) (3) より

$$\int_{-3}^2 (2x+6)^4\,dx = \left[\frac{1}{10}(2x+6)^5\right]_{-3}^2 = 10000$$

(5) $\displaystyle\int \left(x^2+\frac{1}{x^2}\right)\,dx = \frac{1}{3}x^3-\frac{1}{x}$

(6) (5) より

$$\int_1^2 \left(x^2+\frac{1}{x^2}\right)\,dx = \left[\frac{1}{3}x^3-\frac{1}{x}\right]_1^2 = \frac{7}{3}+\frac{1}{2} = \frac{17}{6}.$$

(7) $\displaystyle\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{1+\frac{1}{2}}}{1+\frac{1}{2}} = \frac{2}{3}\,x^{\frac{3}{2}}$

(8) (7) より $\displaystyle\int_0^1 \sqrt{x}\,dx = \left[\frac{2}{3}\,x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}.$

(9) $\displaystyle\int \sqrt{2x+1}\,dx$,

$2x+1=t$ とおくと $\dfrac{dt}{dx}=2$ すなわち $dx=\dfrac{dt}{2}$ だから

$$\int \sqrt{2x+1}\,dx = \int \sqrt{t}\,\frac{dt}{2} = \frac{1}{2}\int \sqrt{t}\,dt = \frac{1}{2}\frac{2}{3}t^{\frac{3}{2}} = \frac{1}{3}(2x+1)^{\frac{3}{2}},$$

(10) (9) より

$$\int_0^4 \sqrt{2x+1}\,dx = \left[\frac{1}{3}(2x+1)^{\frac{3}{2}}\right]_0^4 = 9-\frac{1}{3} = \frac{26}{3}$$

(11) $\displaystyle\int x\sqrt{2x+1}\,dx$,

(9) より $\sqrt{2x+1} = \left(\dfrac{1}{3}(2x+1)^{\frac{3}{2}}\right)'$ だから部分積分法により

$$\begin{aligned}\int x\sqrt{2x+1}\,dx &= \int x\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}\right)'dx \\ &= x\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}\right) - \int\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}\right)dx \\ &= x\left(\frac{1}{3}(2x+1)^{\frac{3}{2}}\right) - \frac{1}{3}\int(2x+1)^{\frac{3}{2}}\,dx\end{aligned}$$

ここで (10) と同様な方法で $\displaystyle\int(2x+1)^{\frac{3}{2}}\,dx = \frac{1}{5}(2x+1)^{\frac{5}{2}}$ が分かるので

$$\begin{aligned}&= \frac{1}{3}x(2x+1)^{\frac{3}{2}} - \frac{1}{15}(2x+1)^{\frac{5}{2}} \\ &= \frac{1}{15}(3x-1)(2x+1)^{\frac{3}{2}}\end{aligned}$$

または (9) と同じ変数変換で

$$\begin{aligned}\int x\sqrt{2x+1}\,dx &= \int \frac{t-1}{2}\sqrt{t}\,\frac{dt}{2}\\ &= \frac{1}{4}\int t^{\frac{3}{2}} - t^{\frac{1}{2}}\,dt\\ &= \frac{1}{10}(2x+1)^{\frac{5}{2}} - \frac{1}{6}(2x+1)^{\frac{3}{2}}\\ &= \frac{1}{15}(3x-1)(2x+1)^{\frac{3}{2}}\end{aligned}$$

(12)　(11) より

$$\int_0^4 x\sqrt{2x+1}\,dx = \left[\frac{1}{15}(3x-1)(2x+1)^{\frac{3}{2}}\right]_0^4 = \frac{121}{5} - \frac{13}{3} = \frac{298}{15}$$

(13) $\displaystyle\int \frac{dx}{x} = \log x$

(14)　(13) より

$\displaystyle\int_1^e \frac{dx}{x} = \Big[\log x\Big]_1^e = \log e - \log 1 = 1 - 0 = 1$

(15) $\displaystyle\int e^x\,dx = e^x$

(16) $\displaystyle\int_0^1 e^x\,dx = \Big[\ e^x\ \Big]_0^1 = e^1 - e^0 = e - 1$

(17)　(9) と同じ変数変換で

$\displaystyle\int e^{2x+1}\,dx = \int e^t\,\frac{dt}{2} = \frac{1}{2}\int e^t\,dt = \frac{1}{2}e^t = \frac{1}{2}e^{2x+1}$

(18) $\displaystyle\int_0^1 e^{2x+1}\,dx = \left[\frac{1}{2}e^{2x+1}\right]_0^1 = \frac{1}{2}\left(e^3 - e\right)$

(19) $\displaystyle\int \sin x\,dx = -\cos x$

(20) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\,dx = \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} - (-\cos 0) = 1$

(21) $\displaystyle\int_0^{\pi} \sin x\,dx = \Big[-\cos x\Big]_0^{\pi} = -\cos\pi - (-\cos 0) = 2$

(22) $2x = t$ とおくと $\dfrac{dt}{dx} = 2$ すなわち $dx = \dfrac{dt}{2}$ だから

$$\int \sin(2x)\,dx = \int \sin t\,\frac{dt}{2} = \frac{1}{2}\int \sin t\,dt = -\frac{1}{2}\cos t = -\frac{1}{2}\cos(2x)$$

(23) $\displaystyle\int_0^{\frac{\pi}{2}} \sin 2x\,dx = \left[-\frac{1}{2}\cos(2x)\right]_0^{\frac{\pi}{2}} = -\frac{1}{2}\left(\cos\pi - \cos 0\right) = 1$

(24) $\displaystyle\int_0^{\pi} \sin 2x\,dx = \left[-\frac{1}{2}\cos(2x)\right]_0^{\pi} = -\frac{1}{2}\left(\cos 2\pi - \cos 0\right) = 0$

(25) $\displaystyle\int \cos x\,dx = \sin x$

(26) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\,dx = \Big[\sin x\Big]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - \sin 0 = 1$

(27) $\displaystyle\int_0^{\pi} \cos x\,dx = \Big[\sin x\Big]_0^{\pi} = \sin\pi - \sin 0 = 0$

(28) $2x = t$ とおくと $\dfrac{dt}{dx} = 2$ すなわち $dx = \dfrac{dt}{2}$ だから

$$\int \cos(2x)\,dx = \int \cos t\,\frac{dt}{2} = \frac{1}{2}\int \cos t\,dt = \frac{1}{2}\sin t = \frac{1}{2}\sin(2x)$$

(29) $\displaystyle\int_0^{\frac{\pi}{2}} \cos 2x\,dx = \left[\frac{1}{2}\sin(2x)\right]_0^{\frac{\pi}{2}} = -\frac{1}{2}\left(\sin\pi - \sin 0\right) = 0$

$$(30)\ \int_0^{\pi} \cos 2x \, dx = \left[\frac{1}{2}\sin(2x)\right]_0^{\pi} = -\frac{1}{2}\left(\sin 2\pi - \sin 0\right) = 0$$