電気のための線形代数 B 演習問題 No.6	学生番号 □□□□□□□		

6.1. $(P)\begin{cases} x_1 - x_2 + 3x_3 = 0 \\ 2x_1 + 3x_2 - 4x_3 = 1 \\ x_1 - 2x_2 + x_3 = 1 \end{cases}$ をクラメルの公式を用いて解きたい.

(1)

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ 1 & -2 & 1 \end{pmatrix}, \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}, \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

とおくと $(P) \Leftrightarrow A\vec{x} = \vec{b}$ となる.

(2) $$|A| = \begin{vmatrix} 1 & -1 & 3 \\ 2 & 3 & -4 \\ 1 & -2 & 1 \end{vmatrix} \begin{matrix} \\ \text{第 1 行を 2 倍して引く} \\ \text{第 1 行を引く} \end{matrix} = \begin{vmatrix} 1 & -1 & 3 \\ 0 & 5 & -10 \\ 0 & -1 & -2 \end{vmatrix} = 1 \times \begin{vmatrix} 5 & -10 \\ -1 & -2 \end{vmatrix}$$

$$= 5 \times (-2) - (-1)(-10) = -10 - 10 = -20$$

$$\triangle_1 = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 3 & -4 \\ 1 & -2 & 1 \end{vmatrix} = -\begin{vmatrix} -1 & 3 \\ -2 & 1 \end{vmatrix} + \begin{vmatrix} -1 & -3 \\ 3 & -4 \end{vmatrix} = -(-1+6) + (4-9) = -5 - 5 = -10$$

$$\triangle_2 = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & -4 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ 2 & -4 \end{vmatrix} = (1-3) - (-4-6) = 8$$

$$\triangle_3 = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -\begin{vmatrix} 1 & -1 \\ 1 & -2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = -(-2+1) + (3+2) = 1 + 5 = 6$$

(3) x_1, x_2, x_3 を求めよ.

$$x_1 = \frac{\triangle_1}{|A|} = \frac{1}{2},\ x_2 = \frac{\triangle_2}{|A|} = -\frac{2}{5},\ x_3 = \frac{\triangle_3}{|A|} = -\frac{3}{10}$$

追加

$$(P)\ \left\{\begin{array}{ccccccc} x_1 & + & 2x_2 & + & 3x_3 & = & 0 \\ x_1 & + & 3x_2 & + & 5x_3 & = & 0 \\ x_1 & + & 5x_2 & + & 12x_3 & = & 1 \end{array}\right. \quad \text{を解く。}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{vmatrix} = 3,$$

$\triangle_1 = 1,$

$\triangle_2 = -2,$

$\triangle_3 = 1,$

だから

$$x_1 = \frac{\triangle_1}{|A|} = \frac{1}{3},\ x_2 = \frac{\triangle_2}{|A|} = -\frac{2}{3},\ x_3 = \frac{\triangle_3}{|A|} = -\frac{1}{3}$$