電気のための微分積分C　第2回解答

2.1.

(1) $\displaystyle\int (2x+6)\,dx = x^2+6x$ だから

$$\int_0^5 (2x+6)\,dx = \Big[x^2+6x\Big]_0^5 = 55.$$

(2) $\displaystyle\int (x^3-3x^2+1)\,dx = \frac{1}{4}x^4 - x^3 + x$ だから

$$\int_1^2 (x^3-3x^2+1)\,dx = \Big[\frac{1}{4}x^4 - x^3 + x\Big]_1^2$$

$$= \left(\frac{1}{4}2^4 - 2^3 + 2\right) - \left(\frac{1}{4}1^4 - 1^3 + 1\right)$$

$$= (4-8+2) - \left(\frac{1}{4}\right) = -\frac{1}{4} - 2 = -\frac{9}{4}.$$

(3) $\displaystyle\int \left(x^2 + \frac{1}{x^2}\right) dx = \frac{1}{3}x^3 - \frac{1}{x}$ だから

$$\int_1^2 \left(x^2 + \frac{1}{x^2}\right) dx = \Big[\frac{1}{3}x^3 - \frac{1}{x}\Big]_1^2 = \frac{7}{3} + \frac{1}{2} = \frac{17}{6}.$$

(4) $\displaystyle\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{1+\frac{1}{2}}}{1+\frac{1}{2}} = \frac{2}{3}\,x^{\frac{3}{2}}$ だから

$$\int_0^1 \sqrt{x}\,dx = \left[\frac{2}{3}\,x^{\frac{3}{2}}\right]_0^1 = \frac{2}{3}.$$

(5) $3x+1=t$　$(*)$ とおく. この両辺を x で微分すると

$$3 = \frac{dt}{dx}$$

となるが, この両辺に $\dfrac{dx}{3}$ を掛けると

$$dx = \frac{dt}{3} \qquad (**)$$

という等式が得られる．(*), (**) によって積分変数を t に置き換えていくと

$$\int \sqrt{3x+1}\,dx = \int t^{\frac{1}{2}}\,\frac{dt}{3} = \frac{1}{3}\frac{t^{1+\frac{1}{2}}}{1+\frac{1}{2}} = \frac{2}{9}\,(3x+1)^{\frac{3}{2}} \text{ だから}$$

$$\int_0^1 \sqrt{3x+1}\,dx = \left[\frac{2}{9}\,(3x+1)^{\frac{3}{2}}\right]_0^1 = \left(\frac{2}{9}\,4^{\frac{3}{2}}\right) - \left(\frac{2}{9}\,1^{\frac{3}{2}}\right) = \frac{14}{9}.$$

(6) $\displaystyle\int \frac{dx}{x} = \log x$ だから

$$\int_1^e \frac{dx}{x} = \Big[\log x\Big]_1^e = \log e - \log 1 = 1 - 0 = 1.$$

(7)

$$\int e^x\,dx = e^x$$

だから

$$\int_0^1 e^x\,dx = [e^x]_0^1 = e^1 - e^0 = e - 1.$$

(8) (5) と同じ変換で

$$\int e^{3x+1}\,dx = \int e^t\,\frac{dt}{3} = \frac{1}{3}e^t = \frac{1}{3}e^{3x+1}.$$

したがって

$$\int_0^1 e^{3x+1}\,dx = \left[\frac{1}{3}e^{3x+1}\right]_0^1 = \left(\frac{1}{3}e^4\right) - \left(\frac{1}{3}e^1\right) = \frac{1}{3}(e^4 - e).$$

(9) $\displaystyle\int \sin x\,dx = -\cos x$ だから

$$\int_0^{\frac{\pi}{2}} \sin x\,dx = \Big[-\cos x\Big]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} - (-\cos 0) = 1.$$

(10) $2x = t$ とおくと (5) と同様にして $dx = \dfrac{dt}{2}$ であるから

$$\int \sin 2x\,dx = \int \sin t\,\frac{dt}{2} = -\frac{1}{2}\cos t = -\frac{1}{2}\cos 2x.$$

だから

$$\int_0^{\frac{\pi}{2}} \sin 2x\, dx = \Big[-\frac{1}{2}\cos 2x\Big]_0^{\frac{\pi}{2}} = -\frac{1}{2}\cos\pi - (-\frac{1}{2}\cos 0) = 1.$$

(11) $\displaystyle\int_1^4 \sqrt{x}\, dx$

(4) と同様に

$$\int_1^4 \sqrt{x}\, dx = \left[\frac{2}{3}x^{\frac{3}{2}}\right]_1^4 = \frac{2}{3}\times 4^{\frac{3}{2}} - \frac{2}{3}\times 1 = \frac{14}{3}.$$

(12) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x\, dx$

$\displaystyle\int \cos x\, dx = \sin x$ だから

$$\int_0^{\frac{\pi}{2}} \cos x\, dx = \Big[\sin x\Big]_0^{\frac{\pi}{2}} = \sin\frac{\pi}{2} - (\sin 0) = 1.$$

(13) $\displaystyle\int \cos 2x\, dx = \frac{1}{2}\sin 2x$ だから

$$\int_0^{\pi} \cos 2x\, dx = \left[\frac{1}{2}\sin 2x\right]_0^{\pi} = \frac{1}{2}\sin 2\pi - \frac{1}{2}\sin 0 = 0 - 0 = 0$$

(14) $\displaystyle\int \cos 2x\, dx = \frac{1}{2}\sin 2x$ だから

$$\int_0^{\frac{\pi}{2}} \cos 2x\, dx = \left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{2}} = \frac{1}{2}\sin\pi - \frac{1}{2}\sin 0 = 0 - 0 = 0$$